

Computer simulations of biaxial nematics

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2008 J. Phys.: Condens. Matter 20 463101

(<http://iopscience.iop.org/0953-8984/20/46/463101>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 129.252.86.83

The article was downloaded on 29/05/2010 at 16:20

Please note that [terms and conditions apply](#).

TOPICAL REVIEW

Computer simulations of biaxial nematics

Roberto Berardi, Luca Muccioli, Silvia Orlandi, Matteo Ricci and Claudio Zannoni

Dipartimento di Chimica Fisica e Inorganica, and INSTM-CRIMSON, Università di Bologna, Viale Risorgimento 4, 40136 Bologna, Italy

E-mail: roberto.berardi@unibo.it

Received 26 June 2008, in final form 2 September 2008

Published 27 October 2008

Online at stacks.iop.org/JPhysCM/20/463101**Abstract**

Biaxial nematic (N_b) liquid crystals are a fascinating condensed matter phase that has baffled, for more than thirty years, scientists engaged in the challenge of demonstrating its actual existence, and which has only recently been experimentally found. During this period computer simulations of model N_b have played an important role, both in providing the basic physical properties to be expected from these systems, and in giving clues about the molecular features essential for the thermodynamic stability of N_b phases. However, simulation studies are expected to be even more crucial in the future for unravelling the structural features of biaxial mesogens at the molecular level, and for helping in the design and optimization of devices towards the technological deployment of N_b materials. This review article gives an overview of the simulation work performed so far, and relying on the recent experimental findings, focuses on the still unanswered questions which will determine the future challenges in the field.

(Some figures in this article are in colour only in the electronic version)

Contents

1. Experiments	3
2. Order parameters and theories	4
3. Model potentials	6
4. Single-site models	7
5. Multi-site models	9
6. Mixtures	11
7. Concluding remarks	12
Acknowledgments	12
References	12

trirefringent materials, i.e. with three different refractive indexes.

The quest for a truly thermotropic N_b has been aptly named '*the holy grail*' [1] of LC since it has fuelled experimental and theoretical research for more than 30 years, starting from the seminal theoretical papers of Freiser [2] and Straley [3]. This is an interesting scientific case since by relying on idealized models the theoretical predictions first, and later on the computer simulations, have paved the way for the difficult task of actually synthesizing molecules with the desired mesogenic behaviour. While theoretical investigations of LC usually lag behind the discoveries of clever and imaginative synthetic chemists, the specific case of N_b is quite remarkably the opposite. However, it should be pointed out that neither theory nor simulations have yielded a specific molecular design but rather have provided reassurance on the N_b not being *a priori* forbidden. The N_b phase was not the only '*missing link*' in the class of thermotropic LC organizations, and as a side comment we also quote the ferroelectric nematic among the elusive phases predicted by theory [4] and computer simulations [5] which is yet an open challenge on the experimental ground. As we shall discuss

Introduction

Biaxial nematic (N_b) liquid crystals (LC) are nematics where the mesogenic molecules organize with two mutually orthogonal directions of preferential alignment (directors), while keeping a uniform distribution of centres of mass (see figure 1). The principal director \mathbf{n} is typical of uniaxial nematics (N_u), while the secondary director \mathbf{m} is specific to biaxial systems which then behave optically as macroscopic

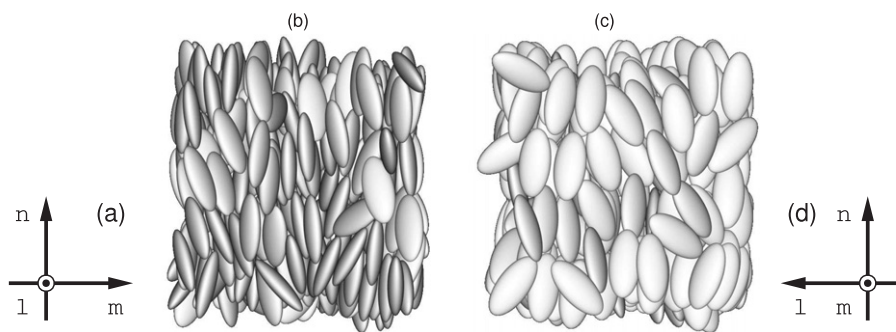


Figure 1. Detail of a N_b phase obtained from the MC simulations of biaxial GB ellipsoids of [53], and showing the principal \mathbf{n} and secondary \mathbf{m} directors. The two snapshots are relative to the same sample observed along \mathbf{l} (plates (a), and (b)), and along \mathbf{m} (plates (c), and (d)).

later, part of the elusivity of N_b may be due to the fact that many of the molecular features that favour a biaxial arrangement of the molecules in the fluid nematic also favour their packing in the competing smectic or crystalline biaxial phases.

The motivations for this search have certainly been widespread, ranging from purely academic interest in an unsolved scientific problem to the potential usage of these materials in faster displays, where in principle the commutation of the secondary director should give lower response times compared to the conventional twisted nematic and ferroelectric smectic devices.

At the time of writing a few instances of stable thermotropic biaxial nematics and their characterization [6–10] have been reported. The papers of Luckhurst [11] and Praefcke [12] and the book of Chandrasekhar [13] also give a critical account of the early synthetic attempts, while assessment of the more recent achievements can be found in [14]. These experimental findings are now fuelling a target oriented research towards other mesogenic compounds, and the implementation of envisaged technological applications. It should be noted anyway that the experimental identification of N_b phases is so difficult that an active debate that includes challenging published results [11, 15, 16] has taken place, and to some extent still is [17]. The role that confinement, external fields and boundaries [18], or sample preparation [19, 20] may play in determining the actual observed biaxial behaviour is also questioned. While ordinary N_u are typically formed by elongated (calamitic, or rod-like), or squashed (discotic, or disc-like) mesogens, what molecular shape is more conducive towards N_b behaviour is far from obvious.

While the search of N_b formed by rod-like and bent-core (boomerang) molecules has been actively pursued, the actual existence of such phases also from disc-like mesogens as predicted by theoretical models [2, 3] and simulations [21–23] is still an open question, and has received little attention from the point of view of chemical synthesis. One possibility that has been suggested [24] is that of combining the properties of rods and discs. The simplest possibility is that of mixing them, but it has been found by theory [25] and experiment [26] that these mixtures tend to phase separate. A recent experimental paper by Apreutesei and Mehl [27] challenges the past modelling work in view of considering the role of flexibility and attractive interactions. Even though LC phase transitions

can be driven by entropic effects [4], the detailed balance of contributions leading to the formation of specific organizations also includes energetic factors arising from attractive–repulsive interactions between molecules.

The chemistry of synthesized mesogens has been even more far-reaching in scope than theories, and besides organic compounds with a rigid aromatic core and terminal flexible alkyl spacers, the ingenuity and rational design of synthetic chemists [28–30] has also considered as candidate N_b mesogens: dendrimers, silicon organic, and metallorganic coordination compounds [31], to name a few.

This paper reviews computer simulations of N_b , and it is fair to say that the field of computer simulations of LC systems has now reached a well established state. Simulations allow one to draw direct relations between specific molecular properties and macroscopic mesogenic behaviour, and to study the spontaneous formation of liquid crystalline phases, characterize their structure, and determine anisotropic properties and responses to external fields. Furthermore, computer simulations automatically account for the n -body correlations in condensed phases, and as such are useful for validating the predictions of theories. We should recall, for instance, that the original predictions for the existence of N_b [2, 3] were based on a simple mean field theory, which is known to make rather large errors in determining phase boundaries. The progress in the field encompasses all classes of lattice, coarse-grained, molecular, and atomistic models, and has been thoroughly reviewed in recent years [32–35], and also presented in NATO schools [36] and workshops [37]. This manuscript relies on these general works and focuses on the specific aspects of simulating N_b systems, which have only partly been discussed in the previous reviews. Section 1 contains a few highlights to the most recent synthetic and experimental findings related to the search for N_b phases. The order parameters used to characterize the biaxial ordering are presented in section 2 along with theoretical models, while the principal potentials used for N_b systems are described in section 3. The remaining sections are specifically devoted to simulation results, and are organized according to the symmetry/structure of the simulation models. Section 4 deals with systems with spins and single-site particles of D_{2h} symmetry. Multi-site potentials for molecules with symmetry C_{2v} or lower, and atomistic models are treated instead in Section 5. The rod–disc mixtures, which are also candidates

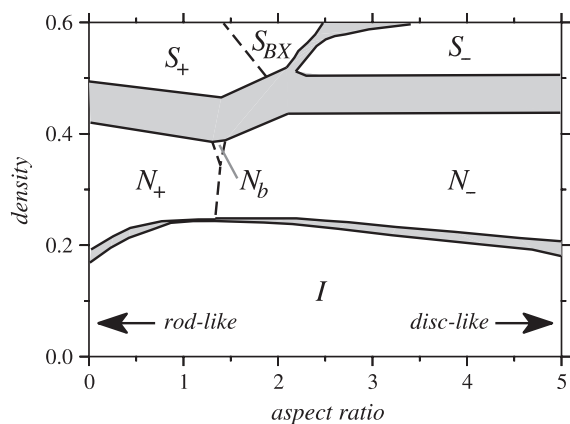


Figure 2. Phase diagram, with respect to the dimensionless density and aspect ratio, from the theoretical spheroplatelets model of [38]. First- and second-order equilibrium curves are plotted as continuous and dashed lines. Coexistence regions are shaded. (Adapted from figure 4 of [38].)

for N_b systems, but still lack conclusive confirmation or disproof, are discussed in section 6. Some concluding remarks close the review.

1. Experiments

From a purely thermodynamical point of view, the most frequent experimental scenario with (virtually) biaxial mesogens is that upon cooling the N_u phase, the free energy of the underlying smectic/columnar becomes lower earlier than that of the N_b phase, which is consequently not observed [38] (see figure 2). The quest for the N_b phase has then been a synthetic (and modelling) attempt of either destabilizing smectic phases, or stabilizing the N_b phase (or both), in such a way that the N_b becomes the equilibrium phase over a finite temperature range. In this respect theoretical models based on purely orientational potentials and lattice simulations are not haunted by the formation of a competing layered or crystal phase before a N_b becomes thermodynamically stable, since they do not consider positional degrees of freedom (see [38] for an exception). This has been turned into an advantage for the identification of the intrinsic molecular properties (e.g. shape anisotropy) relevant for the stabilization of an overall N_b organization, and for efficiently mapping phase diagrams.

It should be noted that our description of N_b phases in terms of a locally uniform distribution of the secondary \mathbf{m} director might be an exception in real systems. Some evidence from experimental measurements and computer simulations [39], and theoretical models [18] is pushing forward a different picture of what the standard N_b phase might be. According to these sources, biaxial or even polar cybotactic clusters might be the most common signs of N_b phases, and the long-range biaxial ordering would be induced by external perturbations (e.g. a field), surface anchoring [40], or shear stress [31]. One of the questions which still awaits an answer is therefore if the observed biaxiality is an intrinsic property of the nematic fluid or instead, as pointed out by Vanakaras and Photinos, a gigantic response

of biaxial cybotactic clusters to the surface anchoring or external field [18]. According to this picture, the discordant measurements of phase structure for claimed N_b might also have originated from the boundary conditions typical of each experiment, and the ensuing different degrees of alignment of biaxial clusters. These issues are also related to the observation of N_u phases with cybotactic clusters of Torgova *et al* [41], or the spontaneous segregation into chiral domains reported by Görtz and Goodby [19] and Bruce and coworkers [20]. For this latter example, the formation of organized supramolecular structures seems to take place above the ordering transition, and it also depends on the sample history, hinting at a kinetic rather than thermodynamic effect.

The experimental identification of biaxiality has always proved to be fairly difficult [16, 42, 43], and still does because of the small magnitude of the transversal orientational ordering, and the necessity of filtering out the possible perturbations arising from the laboratory setup and anchoring conditions [43]. Actually, this is an additional hurdle to cope with for those scientists trying to characterize potentially N_b systems. As a matter of fact, most claims of N_b phases published earlier than 2004 seem to have been caused by deceptive experimental evidence, later challenged by independent measurements of different anisotropic properties.

The first convincing thermotropic biaxial nematic phase was jointly reported in year 2004 by Kumar, Samulski and coworkers [6–8] from bent-core mesogens based on a mesogenic oxadiazole core [6] with lateral substituents. The phase has been further characterized by x-ray [7, 44], polarized microscopy, conoscopy, and deuterium NMR [8], and also by polarized Raman spectroscopy [45] providing independent experimental evidence of the N_b organization. The interpretation of these results has anyway been questioned [17], but an answer from the authors [46] dismissed the doubts raised. Even if the N_b for these mesogens occurs at high temperature ($\approx 200^\circ\text{C}$) these molecules have opened up the search for other systems forming N_b phases under operating conditions closer to room temperature, as needed for standard technological applications. Kumar and coworkers have reported another class of rigid bent-core mesogens, with aperture angle of 90° [47], which appear to also form N_b phases.

This thermotropic N_b behaviour is far from being the common one for bent-core mesogens, which are often devoid of a N_u phase. Even molecules with a chemical structure very similar to that of the compounds studied by Kumar and Samulski do not have a N_b phase but display other interesting properties, such as the formation of nematic phases with cybotactic clusters [41] stable over a quite wide range of temperatures.

Besides the bent-core-shaped mesogens, in the same year 2004, a novel class of N_b based on organo-siloxane tetrapodes was found by Mehl, Vij and coworkers [9]. These thermotropic systems have been thoroughly characterized by measuring the anisotropy of infrared absorbance, with the addition of conoscopy and textural imaging under polarized light [9], and the temperature dependence of the averaged quadrupolar coupling constant of a deuterated 8CB solute

probe by deuterium NMR [10]. Such tetrapodes are quite complex and for the time being represent a challenge for a computer simulation aimed at understanding the molecular origin of their N_b phase, especially in relation to their high flexibility which makes it difficult to define their shape and interaction anisotropies in terms of simple models.

Severing and Saalwächter [48] have also given evidence of a new class of N_b LC polymers with lateral mesogens. This is a subsequent development after the seminal paper of Hessel and Finkelmann [49]. Also new asymmetric bent-core mesogenic compounds forming N_u and N_b phases have been reported [50, 19]. The differential scanning calorimetry measurements for these systems support the theoretical [51] and computer simulation [52, 53] prediction of a second-order transition from N_u – N_b . Also for organo-siloxane tetrapodes [9] quite accurate calorimetric experiments could not detect discontinuities across the N_u to N_b transition [54].

N_b phases open the possibility of designing new bistable devices with a faster response than those based on N_u , because the characteristic switching times of the principal and secondary directors are expected to be very different. The dynamics for the electro-optical commutation processes of a N_b phases when confined between planar plates has been measured by Lee *et al* [55], and this is the first experimental investigation studying the issues related to switching experiments of N_b phases (also see the comment of Stannarius [175]). Other experimental studies probing biaxial bistability have been performed, for instance the time resolved characterization of the commutation of a nematic LC cell submitted to a strong electric field (to induce biaxiality) between two topologically distinct textures [56], or with covalently bonded rod- and disc-like mesogens [40]. In spite of these results, the devising of efficient bistable N_b devices relying on the switching of the secondary director might not be easy, and a significant amount of experimental and computer modelling work still has to be done.

2. Order parameters and theories

The anisotropic properties of N_b phases are expected to arise from a long-range degree of orientational ordering along two orthogonal directions, the principal \mathbf{n} and the secondary \mathbf{m} directors, while molecular centres of mass have a random distribution in space. The quantitative assessment of this organization is conventionally made by measuring (or computing) suitable orientational order parameters. The symmetry analysis of the D_{2h} N_b phases identifies a set of four different second rank order parameters that can be used to fulfil the task.

Unfortunately, there is no universally adopted convention about the definition and notation of such order parameters. Over the years a number of equivalent sets have been used, and the paper of Rosso [57] gives a quite complete list of the notations employed for these order parameters, and is certainly useful in decoding the various conventions and comparing the published findings. The two most popular definitions are those based on a Cartesian representation of the order matrices [22, 58], and those using symmetrized Wigner

matrices [21]. The first set arises from a purely mathematical modelling of the alignment process, and has the (nice) feature that all order parameters range between 0 (no ordering) and 1 (complete ordering). The second definition originates from the formal description of static physical observables (e.g. from NMR, Raman, or fluorescence depolarization measurements) obtained from the experimental characterization of LC phases. This formulation relies on an irreducible tensors approach [59], where the order parameters are average values of the Wigner rotation matrices transforming from the laboratory to director frame (and vice versa). The two sets of order parameters are equivalent and mathematical relations convert from one to the other. We quote here the second kind of definition, using the scalar products between molecular and director frame axes to identify rotations. The equivalent definition in terms of Euler angles can be found in [21]. The Wigner matrices symmetrized for the D_{2h} group of a N_b phase are

$$R_{m,n}^L = \frac{1}{4} \delta_{m,\text{even}} \delta_{n,\text{even}} [D_{m,n}^{L*} + D_{-m,n}^{L*} + D_{m,-n}^{L*} + D_{-m,-n}^{L*}]. \quad (1)$$

The order in a N_b phase, and the mean values of second rank tensorial observables, can be characterized with the following ensemble averages

$$\langle R_{0,0}^2 \rangle = \langle \frac{3}{2} (\mathbf{z} \cdot \mathbf{n})^2 - \frac{1}{2} \rangle, \quad (2)$$

$$\langle R_{2,0}^2 \rangle = \left\langle \sqrt{\frac{3}{8}} [(\mathbf{z} \cdot \mathbf{l})^2 - (\mathbf{z} \cdot \mathbf{m})^2] \right\rangle, \quad (3)$$

$$\langle R_{0,2}^2 \rangle = \left\langle \sqrt{\frac{3}{8}} [(\mathbf{x} \cdot \mathbf{n})^2 - (\mathbf{y} \cdot \mathbf{n})^2] \right\rangle, \quad (4)$$

$$\langle R_{2,2}^2 \rangle = \langle \frac{1}{4} [(\mathbf{x} \cdot \mathbf{l})^2 - (\mathbf{x} \cdot \mathbf{m})^2 - (\mathbf{y} \cdot \mathbf{l})^2 + (\mathbf{y} \cdot \mathbf{m})^2] \rangle, \quad (5)$$

where \mathbf{l} is the third axis of a (right-handed) Cartesian frame defined by the first two. The molecular axes are instead \mathbf{x} , \mathbf{y} , and \mathbf{z} .

Typical ranges for these order parameters in models and simulations of N_u and N_b phases are approximately [0.4, 0.8] for $\langle R_{0,0}^2 \rangle$ and [0.1, 0.3] for $\langle R_{2,2}^2 \rangle$ (see [23] and figure 9 for an exception). An example of temperature dependence of such order parameters from a lattice simulation of a N_b is given in figure 4. It should be noted that molecular models (both hard and soft attractive–repulsive) usually overestimate these values, especially at the ordering transitions. Experimental values are usually much smaller, in particular for the observed biaxialities $\langle R_{2,2}^2 \rangle$. Differently from molecular models which offer at most semi-quantitative results, the atomistic simulations with predictive capabilities [60, 34, 61] can provide reliable estimates of such order parameters as well as other macroscopic properties.

Whilst widely adopted, the sets of second rank order parameters are not sufficient to characterize all LC phases, either known or theoretically envisaged. For instance, bent-core mesogens can form an extremely rich class of LC and their proper characterization also requires third rank tensors [4]. Another counterexample is that of ferroelectric phases where also first rank order parameters are relevant [62, 63]. More generally, order parameters of rank higher than two are necessary for characterizing the orientational distribution [58],

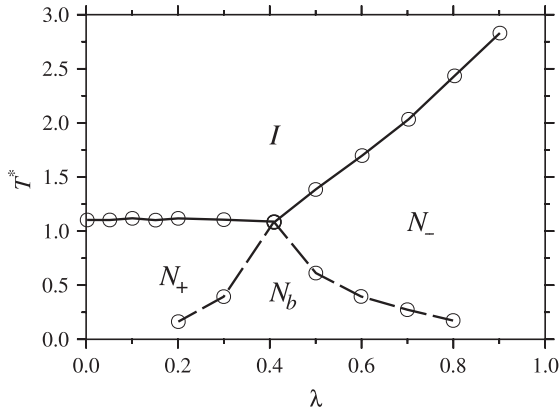


Figure 3. Phase diagram, with respect to dimensionless temperature T^* and shape biaxiality λ (see equation (6)), from the MC simulations of [21] using a $10 \times 10 \times 10$ square lattice and the potential of equation (7). First and second-order equilibrium curves are plotted as continuous and dashed lines. (Adapted from figure 2 of [21].)

and can be obtained from some experimental techniques. For instance, polarized Raman spectroscopy has been used to measure, for the first time, the order parameters of fourth rank [45] in the N_b of [7, 8]. On a similar level, the interpretation of experiments probing the molecular reorientation in N_b (e.g. Raman, fluorescence depolarization, NMR, dielectric relaxation), requires biaxial time correlation functions, that have been evaluated by solving a generalized rotational diffusion equation for both a uniaxial [64] or a biaxial [65] molecule reorienting in a biaxial phase.

Prior to computer simulations, laboratory syntheses and experiments, the theoretical models have provided the general framework background information for all subsequent studies on N_b systems. The fundamental proof of principle for the possible existence of a thermotropic nematic phase with three orthogonal optical axes, was given by Freiser [2] first, and Straley [3] a few years later. By considering mesogens as rigid board-like particles (sometimes called sanidic) it was found that shape biaxiality λ (which is zero for cylindrical $D_{\infty h}$ symmetry, i.e. rod-like and disc-like particles) is the physically relevant parameter connected to the excluded volume

$$\lambda = \sqrt{3/2} \frac{\sigma_x - \sigma_y}{2\sigma_z - \sigma_x - \sigma_y}, \quad (6)$$

where σ_x , σ_y , and σ_z are the particle dimensions. The mean field models have phase diagrams where both rod- and disc-like parameterizations show a transition from isotropic to N_u (conventionally labelled N_+ for calamitic, and N_- for discotic nematics, also see figure 3), followed by a second one to N_b [66]. At the crossover point (also named the Landau point) the molecular biaxiality λ becomes maximum, and the ordering transition from isotropic leads directly to the N_b phase (see figure 3). Since this state point is the one where the N_b phases were expected to appear at the highest temperature, the synthetic chemists focused toward assembling mesogenic molecules with an effective shape biaxiality falling within this region. At the Landau point biaxiality identifies the particles as rod-like and disc-like at the same time. However, by using

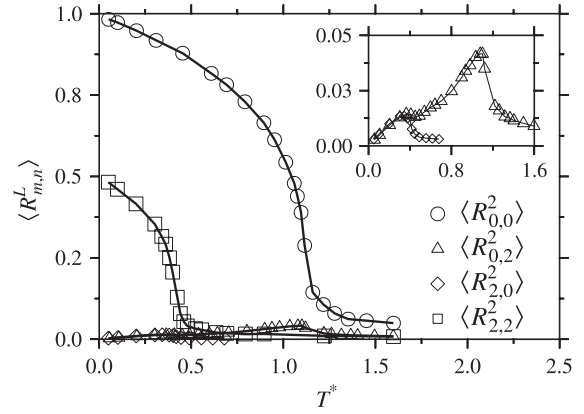


Figure 4. The average order parameters $\langle R_{m,n}^2 \rangle$ (see equations (2)–(5)), plotted against dimensionless temperature T^* , for a shape biaxiality $\lambda = 0.3$ of figure 3. (Adapted from figures 3 and 4 of [21].)

scaled particle theory and also taking into account translational order Taylor and Herzfeld [38] have shown how in a fluid of rigid spheroplatelets the range of existence for the N_b phase becomes extremely narrow (see figure 2), or even vanishes, when a smectic organization can form (as usually observed).

The effect of polydispersity or variable shape in model single-component systems have been accounted for by either considering a frequency-dependent polarizability anisotropy [67], or with a Gaussian distribution of shape biaxiality modelled as a quadrupolar mass distribution [68]. For two-component systems this effect is also quite relevant since it may be a pathway for the stabilization of N_b phases by preventing demixing in mixtures, and it has been studied theoretically by Ratón and Cuesta [69] (see figure 6), and experimentally by van der Kooij and Lekkerkerker [26]. Another interesting result is the paper of Biscari *et al* [70] which discusses the possibility of inducing an N_b phase by using curved surfaces with homeotropic anchoring. The recent analysis of Vanakaras and Photinos [18] suggests to us that our view of N_b phases should be widened to include macroscopic uniaxial systems with a large transversal response where a field-induced biaxiality could be used for bistable devices. From this point of view, earlier mesogens which have been dismissed as forming N_b phases might still have interesting response properties due to the presence of locally biaxial cybotactic clusters, especially if they possess a negative dielectric anisotropy [18]. Over the last few years other theoretical models and studies of N_b have been published [71–76] showing how the field of N_b is quite lively within the scientific community.

Models (and simulations) have been extensively used to study distributions of the nematic directors and topological defects [13, 77–79, 42, 80] in N_b . Theories for the elastic [81–88, 41], flexoelectric [89, 90], and rheological [91] behaviour of N_b have also been proposed. However, computer simulations of the associated mesoscopic coefficients (e.g. elastic constants, viscosities) for N_b have not been performed systematically. These coefficients are also necessary for the description of N_b fluids using mesoscopic

models, or finite elements methods and further work is required in the future for the reliable prediction of these relevant quantities.

3. Model potentials

Several classes of potentials have been used in the attempt to model samples of biaxial particles, and it is somewhat mystifying that while most N_b computer simulations have considered single-site D_{2h} models it has instead been the class of bent-core mesogens which provided the first experimental evidence of a thermotropic N_b phase. In this section we review the principal types of potentials in order of increasing complexity, from lattice to atomistic models, postponing the discussion of the simulation results to the following sections.

Due to their computational cheapness, lattice models were among the first ones to be used by using suitable anisotropic orientational potentials between neighbouring sites. Among the many lattice models which have been considered in computer simulations, the most studied so far is the biaxial generalization to D_{2h} symmetry of the Lebwohl–Lasher potential [66]

$$U(\omega_{12}) = -\epsilon_0(R_{0,0}^2(\omega_{12}) + 2\lambda_{0,2}[R_{0,2}^2(\omega_{12}) + R_{2,0}^2(\omega_{12}) + 4\lambda_{2,2}R_{2,2}^2(\omega_{12})]), \quad (7)$$

where ω_{12} is the relative orientation of neighbouring sites on a cubic lattice. An equivalent expression can be cast in Cartesian form as [52]

$$U(\omega_{12}) = -\epsilon_0(\frac{3}{2}V_{z,z} - \sqrt{6}\lambda_{0,2}[V_{x,x} - V_{y,y}] + 2\lambda_{2,2}^2[V_{x,x} + V_{y,x} - V_{x,y} - V_{y,x}] - \frac{1}{2}), \quad (8)$$

where $V_{a,b} = (\mathbf{a}_1 \cdot \mathbf{b}_2)^2$, and the unit vectors \mathbf{a}_1 , \mathbf{b}_2 are the axes \mathbf{x}_i , \mathbf{y}_i , and \mathbf{z}_i of two neighbouring lattice sites. In the case of dispersive interactions the model parameters $\lambda_{0,2} = \lambda$, and $\lambda_{2,2} = \lambda^2$ are both defined in terms of a biaxiality parameter λ . Furthermore, both λ and ϵ_0 are related to the anisotropy of the polarizability tensor [66, 52]. This is a particular case of the archetypal formulation of Straley [3], where the parameters $\lambda_{m,n}$ are independent and can be chosen to model more general interaction schemes. For instance, the mesogenic properties for a different choice $\lambda_{0,2} = \lambda$, and $\lambda_{2,2} = 0$ have been studied theoretically and with MC simulations in [67, 75]. Lattice models have been thoroughly studied and are now mostly useful for investigating mesoscopic properties of fairly large samples, such as topological defect distributions and optical properties [92].

A second broad class of potentials used in simulations is that of molecular models, either purely repulsive (e.g. hard ellipsoids [93, 22], or hard spherocylinders [94–96]), or attractive–repulsive (e.g. Gay–Berne [97]). These potentials allow one to draw a clear link between specific molecular properties, in particular shape and interaction anisotropies, and collective mesogenic behaviour. For instance since the work of Allen [22], the hard-ellipsoid fluid has been thoroughly studied considering how the semiaxes a , b , and c , and the aspect ratios $a:b:c$ determine the phase diagram. Several off-lattice soft potentials have been derived from the Gaussian overlap model due to Berne and Pechukas [98]. The first one was

that of Ayton and Patey [99] who proposed a generalization for describing purely repulsive soft biaxial ellipsoidal particles. By reducing the thickness of a uniaxial 1:1:3 ellipsoid (which as a hard particle was known to be devoid of a nematic phase) to a 0.8:1:3, and then to a 0.4:1:3 aspect ratio, first a N_u and then a N_b were found (although for a single state point). Unfortunately the study of this model has been discontinued.

The most studied molecular systems are those belonging to the class of attractive–repulsive off-lattice potentials obtained by generalizing the Lennard-Jones (LJ) potential to ellipsoidal shape. The standard model of this class is the Gay–Berne (GB) [97]. The interaction between unlike biaxial particles can be written as [100, 101]

$$U(\mathbf{r}, \omega_1, \omega_2) = 4\epsilon_0\epsilon(\mathbf{r}, \omega_1, \omega_2)[u^{12}(\mathbf{r}, \omega_1, \omega_2) - u^6(\mathbf{r}, \omega_1, \omega_2)], \quad (9)$$

where $u(\mathbf{r}, \omega_1, \omega_2) \equiv \sigma_c/(r - \sigma(\mathbf{r}, \omega_1, \omega_2) + \sigma_c)$, depends on the anisotropic contact term $\sigma(\mathbf{r}, \omega_1, \omega_2)$ parameterized in terms of the three axes σ_x , σ_y , and σ_z of the ellipsoid, and the orientations ω_1 , ω_2 for the molecules, and the intermolecular vector \mathbf{r} . The anisotropic interaction term $\epsilon(\mathbf{r}, \omega_1, \omega_2)$ defines the potential well depth and it is parameterized in terms of the three axes σ_i , and three interaction coefficients ϵ_x , ϵ_y , and ϵ_z defining the relative energy for the *side-by-side*, *face-to-face*, and *end-to-end* configurations of a pair of particles [101] (see figure 7). Three additional empirical parameters σ_c , μ , and ν can be tuned to modify the width and depth of the interaction wells. The constant ϵ_0 defines the energy scale. The contact term $\sigma(\mathbf{r}, \omega_1, \omega_2)$ approximates the geometrical ‘contact distance’ between two ellipsoids (see [102, 103] for a discussion). A similar generalization of the GB potential to non-homogeneous biaxial interactions is that of Cleaver *et al* [104]. The issues concerning the approximation of the contact distance for ellipsoids using the recipe of [98] (i.e. $\sigma(\mathbf{r}, \omega_1, \omega_2)$) have stimulated the proposal of additional generalizations of the biaxial GB model such as the RE-squared potential of Ejtehadi and Everaers [102, 105, 106], which is based on Hamaker theory, and that of Paramonov and Yaliraki [107] who have used the elliptic function approach due to Perram and Wertheim [108]. Both biaxial [100, 101] and RE-squared [102] versions of the GB potential have recently escaped the realm of in-house simulation codes to be included into the popular molecular dynamics engine LAMMPS [109, 110] released under the open-source licencing scheme.

These single-site potentials were initially used to simulate mesogens with board-like symmetry, but were also combined to model lower symmetry molecules (e.g. boomerang shaped) in terms of multi-site objects as described later on. In this class of molecular potentials little attention has been given to including and understanding the role of flexibility, probably to keep the models as computationally cheap as possible. We now recognize that these neglected contributions, when taken into account, may also compensate for the unrealistic density changes across ordering transitions which curse most simulations of both hard and soft molecular models, and which are not observed experimentally.

Atomistic potentials account for flexibility implicitly, however, the complexity of candidate mesogens poses

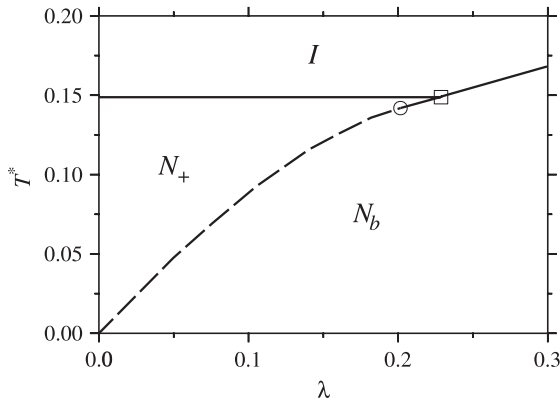


Figure 5. Phase diagram, with respect to the dimensionless temperature T^* and shape biaxiality λ (see equation (6)), from the theoretical model of [118]. First- and second-order equilibrium curves are plotted as continuous and dashed lines. The tricritical and triple points are indicated as a circle and a square. (Adapted from figure 2 of [118].)

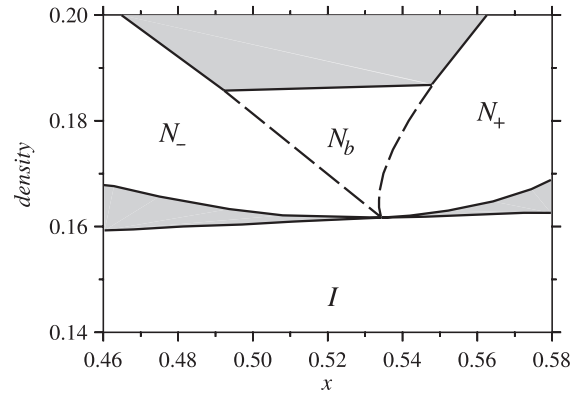


Figure 6. Phase diagram, with respect to the dimensionless number density and mole fraction x of the rod-like mesogen, for a mixture of polydispersed uniaxial rod- and disc-like boards with aspect ratios 1:1:5 and 5:5:1 from the theoretical model of [69]. First- and second-order equilibrium curves are plotted as continuous and dashed lines. Coexistence regions are shaded. (Adapted from figure 2 of [69].)

a challenge since this modelling requires computational resources which have become available only in the past few years. In this class of models the total potential energy is broken down into a sum of intra-molecular terms describing how the energy changes upon variations in bond angles and lengths, conformational motions, and atomic positions, with the addition of intermolecular terms giving non-bonding and electrostatic interactions. A generic atomistic potential can be written as

$$\begin{aligned}
 U_{\text{total}} = & \sum_{\text{bonds}} K_r (r - r_{\text{eq}})^2 + \sum_{\text{angles}} K_\theta (\theta - \theta_{\text{eq}})^2 \\
 & + \sum_{\text{dihed}} \sum_{n=0}^6 V_n [1 + \cos(n\phi + \gamma)] \\
 & + \sum_{\text{atoms}}^{i < j} \left\{ \sqrt{\epsilon_i \epsilon_j} \left[\left(\frac{\sigma_i + \sigma_j}{2r_{ij}} \right)^{12} - \left(\frac{\sigma_i + \sigma_j}{2r_{ij}} \right)^6 \right] \right. \\
 & \left. + \frac{q_i q_j}{r_{ij}} \right\}. \tag{10}
 \end{aligned}$$

The specific mathematical formulation of the additive terms and the complete collection of coefficients K , V , σ , ϵ , and atomic charges q define the force field.

Even if atomistic simulations can provide invaluable details on specific systems, often hardly or not accessible at all by experimental techniques, the task of correctly reproducing phase transition temperatures for a given mesogen is to date not trivial, and this is mainly due to the absence of force fields specifically parameterized for the purpose. Therefore, prior to performing atomistic simulations, a necessary step is that of testing if the chosen force field is adequate: for instance dihedral potentials should be checked as the full conformational space often determines the phase behaviour [111]. In addition, other force field terms may need to be tuned to match relevant experimental observables (e.g. density). If these optimizations are not sufficient (see e.g. [112–115]), a long and computationally demanding reparameterization procedure is needed [116, 115]. We should

also notice that atomistic potentials have been successfully used only recently [61] because no convincing N_b had been reported earlier and molecular structures to start with were not available. From this point of view computer simulations of lattice and molecular models have been invaluable for studying the fundamental properties of N_b systems prior to their experimental discovery.

4. Single-site models

The simplest computer simulation models for N_b are the single-site ones where the mesogenic system is described by a collection of weakly interacting potential centres with orientational degrees of freedom. The interaction sites can be either fixed on a lattice or free to move (off-lattice), and can thus represent an uniformly oriented molecular domain or a single molecule.

Lattice simulations have been the first ones to provide evidence of a spontaneous thermotropic N_b ordering, starting from the MC computer simulations of Luckhurst and Romano [117] who used the potential of equation (7). The complete phase diagram and the temperature dependence of the four second rank biaxial order parameters has been obtained by Biscarini *et al* [21] (see figure 3). These results have been compared against theoretical mean field predictions to find a semi-quantitative agreement between the two (see figure 4). The transition from N_u to N_b has been predicted [51] and simulated [52] with dispersive models to be of second order. There are, in any case, theoretical predictions by Virga and coworkers based on a mean field model that this behaviour is not universal. Relaxing the parameterization constraints of equation (7) two narrow regions of weak first-order transitions between N_u and N_b , and between isotropic to N_b have been found in correspondence of a nematic–nematic tricritical point [67, 118, 72] in the proximity of the Landau point (see figure 5). Recent dynamic light scattering measurements based on organo-siloxane tetrapodes [119], and lattice computer

simulations [120], support these theoretical models. Bates and Luckhurst [121, 122], and Romano [120, 123–125] have further simulated several generalizations of the model potential of equation (7). Lattice models are computationally cheap, and taking advantage of this, it is possible to simulate extensively samples with (sub)-mesoscopic size, and large enough to compute director fluctuations, topological defects structure, and schlieren textures of confined systems or free standing films. These are interesting properties because optical microscopy (either orthoscopy and conoscopy) is often used as a screening technique for quickly identifying LC phases, Chandrasekhar [13, 77] predicted the two-brush defects to be a signature of N_b phases, differently from N_u LC which are characterized instead by four-brush defects. Chiccoli *et al* [79, 42] have theoretically computed the free energy difference between two- and four-brush defects, and produced optical images from MC computer simulations of an N_b lattice model. Such studies have outlined that the presence of topologically stable two-brush defects is not a universal fingerprint of N_b phases because their appearance is related not only to the degree of biaxial orientational ordering but also to the magnitude of the elastic constants. This is a clear example of the usefulness of computer simulations in providing a possible explanation for the earlier claims of N_b mesogens based on deceptive optical measurements.

Moving to off-lattice potentials we quote the seminal MC simulation of Allen [22] who mapped the phase diagram of a model fluid formed by hard ellipsoids with three different axes (aspect ratios ranging from 1:1:10 to 1:10:10). This was the first simulation with full translational and rotational degrees of freedom to compare against the results of theoretical models, and it was a proof of principle that a N_b phase might be thermodynamically more favourable than a smectic, or solid one. For such hard particle model the formation of a N_b phase is based on an entropic driving force related to the excluded volume of ellipsoidal particles. Allen was able to trace the phase diagram with respect to the shape biaxiality finding calamitic N_+ , and discotic N_- nematic phases, and also conducted detailed simulations for the Landau crossover shape. In a later work Camp and Allen [126] studied, with higher accuracy, the phase diagram locating the transition points for the prolate parameterization. The range of shape anisotropies leading to biaxial nematic phases was found to be very narrow, supporting the elusive character of this mesophase. Recently McBride *et al* [127] performed additional Monte Carlo simulations of the hard biaxial ellipsoids fluid studying the reliability of the theoretical equation of state in predicting the isotropic–nematic transition and finding fairly good agreement.

Repulsive off-lattice simulations have been useful for studying the entropic effects stabilizing N_b phases. However, hard particle models do not consider the attractive interactions which are expected to be important in real N_b [61], and which have been found to stabilize the N_b phase for aspect ratios closer to those of conventional mesogens. For instance, uniaxial hard ellipsoids and hard spherocylinders with aspect ratios equal to or smaller than respectively 1:1:3 and 1:1:5 do not form nematic phases upon compression, while soft attractive–repulsive ones do [32].

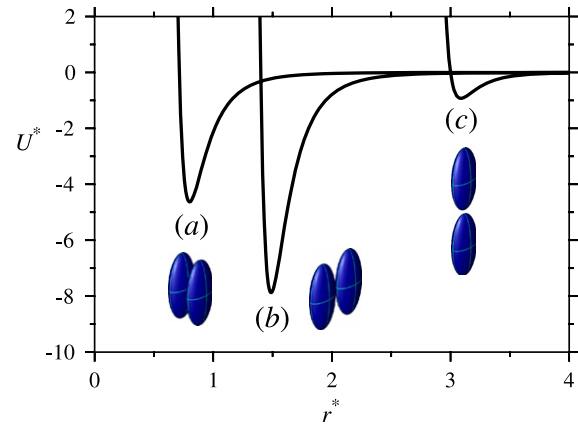


Figure 7. The dimensionless $U^* = U/\epsilon_0$ GB potential profiles for a pair of biaxial ellipsoids in the *face-to-face* (curve a), *side-by-side* (curve b), and *end-to-end* (curve c) configurations [100]. The parameters $\sigma_x = 1.4$, $\sigma_y = 0.714$ and $\sigma_z = 3$ (all in σ_0 units), and $\epsilon_x = 1.7$, $\epsilon_y = 1$ and $\epsilon_z = 0.2$ (all in ϵ_0 units) are those of [53]. Empirical parameters $\mu = 1$, $\nu = 3$, and $\sigma_c = \sigma_y$. Dimensionless distance $r^* = r/\sigma_0$. (Adapted from figure 2 of [53]; we note that the value $\epsilon_y = 1.2$ given at page 5975 of [53] was misprinted, and should be instead $\epsilon_y = 1$.)

The effect of attractive interactions was first considered in a MC simulations of biaxial ellipsoids [53] modelled with the generalized GB potential [100, 101]. These simulations have given hints about the competing role of shape and interaction anisotropies in stabilizing the N_b phase, and have shown that a suitably parameterized off-lattice system with attractive–repulsive soft particles with aspect ratios 1.4:0.714:3, and interactions 1.7:1:0.2, might form a stable thermotropic N_b phase (see figure 8). The choice of opposite shape and interaction biaxialities destabilizes the smectic phase, which is normally observed below the uniaxial nematic phase whenever both shape and interaction anisotropies are positive [13, 128]. The usage of *side-by-side* interactions stronger than those *face-to-face*, i.e. giving a negative interaction biaxiality, produces particles with a dual nature: rod-like from the point of view of shape, and disc-like with respect to interactions [13] (see figure 7). Virtual MD experiments [129] have been used to estimate the switching times of the principal and secondary director of this model N_b [130]. The reorientation of the secondary director \mathbf{m} has been found to be, on average, an order of magnitude faster than that of \mathbf{n} . This kind of direct measurement of a response might be useful for screening the technological suitability of N_b phases in displays and other devices, since in virtual experiment the setup such as sample shape, pair potential, anchoring geometry and strength, and coupling to an external field can be controlled by design without a prior knowledge of mesoscopic response coefficients.

A drawback of all molecular models is that it is not easy to map an idealized potential with specific parameters into a real molecular structure. For instance, in the case of the biaxial ellipsoidal particles of [53], the stronger lateral interactions may be obtained by synthesizing a mesogen with suitable lateral substituents [28] giving weak bonding (e.g. hydrogen bonding groups), or with a specific electrostatic charge distribution. The practical realization of this prescription

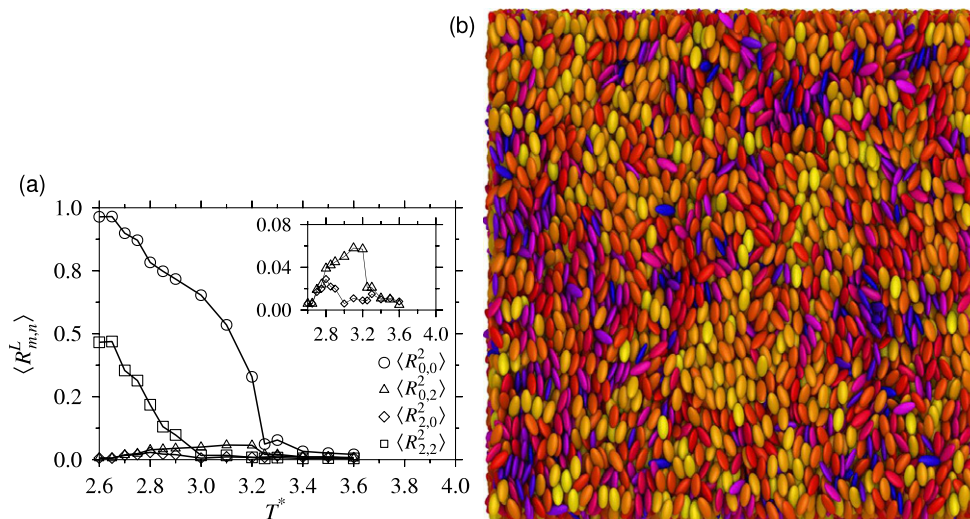


Figure 8. Average order parameters $\langle R_{m,n}^2 \rangle$ (plate (a)) from the NPT MD simulations of the model biaxial rod-like GB mesogen of [53], and snapshot (plate (b)) of a N_b phase formed by 65 536 elongated biaxial GB ellipsoids at $T^* = 2.8$ and $P^* = 8$, with $\langle R_{0,0}^2 \rangle = 0.78$, and $\langle R_{2,2}^2 \rangle = 0.22$. Sample viewed along the \mathbf{m} mesophase director.

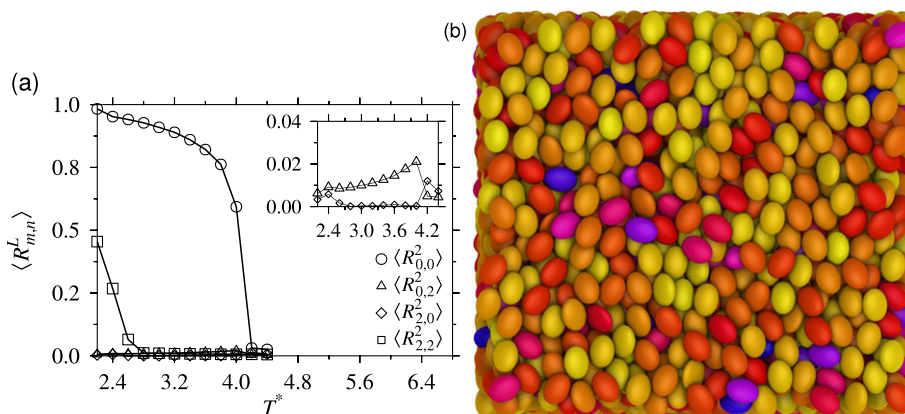


Figure 9. Average order parameters $\langle R_{m,n}^2 \rangle$ (plate (a)) from the NPT MD simulations of the model biaxial disc-like GB mesogen of [23], and snapshot (plate (b)) of a N_b phase formed by 8192 squashed biaxial GB ellipsoids at $T^* = 2.4$ and $P^* = 8$ with $\langle R_{0,0}^2 \rangle = 0.95$, and $\langle R_{2,2}^2 \rangle = 0.27$. Sample viewed along \mathbf{n} mesophase director.

is in any case not straightforward. For instance Bruce and coworkers [131] have ingeniously tailored metallorganic mesogens, although without obtaining N_b mesogens.

Very limited experimental and theoretical work has been done so far for disc-like N_b mesogens, possibly because to date only a few systems have been experimentally found to form a thermotropic nematic phase, the columnar one being their most common anisotropic organization. In [23] a first attempt to study, with MC computer simulations, the competition of shape and interaction anisotropies for biaxial GB discs was presented. A discotic N_b phase was found for GB particles with opposite shape and interaction biaxialities, i.e. having *face-to-face* pair interactions much weaker than those *side-by-side* (see figure 9). The most striking result of these MC simulations was that the columnar phase was completely suppressed in favour of a nematic fluid stable over a quite wide range of temperature, and for order parameter values much higher ($\langle R_{0,0}^2 \rangle > 0.9$) than those typical of a calamitic nematic phase.

5. Multi-site models

Multi-site potentials allow one to model more specifically the effects of molecular shape and interaction anisotropy of mesogenic molecules. The range of simulated systems is fairly large and goes from rigidly connected hard particles to molecular models with charges, dipoles, and quadrupoles, to fully atomistic models with internal degrees of freedom. Asymmetric models have received little attention, even though experimentally have been reported to widen the N_u temperature range [19].

Among board-like models, the papers of Sarman [132, 133] report MD simulations of systems formed by rigid objects obtained by embedding from 8 to 11 repulsive GB discs along a line, with aspect ratios similar to those of the hard ellipsoids of Allen [22]. The compression of isotropic samples of all four model particles gave a first-order transition to the N_u phase (either calamitic N_+ , or discotic N_-), followed by

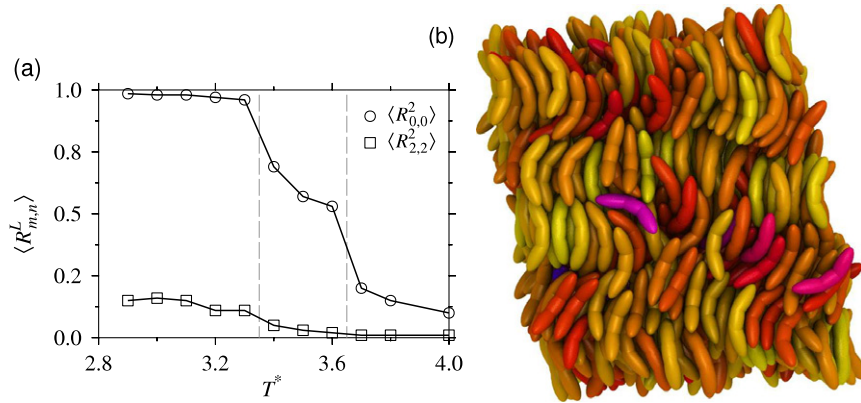


Figure 10. Average order parameters $\langle R_{m,n}^L \rangle$ (plate (a)) from the NPT MC simulations of the model bent-core GB mesogen of [143], and snapshot (plate (b)) of a N_u phase formed by 1000 three-site GB particles with two terminal dipoles at dimensionless temperature $T^* = 3.5$ and pressure $P^* = 10$ and $\langle R_{0,0}^2 \rangle = 0.57$, and $\langle R_{2,2}^2 \rangle = 0.03$. Sample viewed from a direction perpendicular to the \mathbf{n} mesophase director.

transitions to N_b . Sarman [134, 135] has also estimated some viscosities using Green–Kubo relations and linear response theory. This is the only computer simulation of a multi-site molecular model reported so far to yield a N_b . In spite of that, multi-site models have been extensively used to study bent-core mesogens, but generally speaking most of the simulations reported in literature do not provide N_b (actually nematic phases are rather exceptional for such class of molecules, and smectic LC are the typical organizations observed experimentally and in simulations). Since more N_b mesogens with bent-core shape may be found in the future, it is nonetheless interesting to overview some of the published simulation results.

The majority of published simulations for bent-core mesogens are based on rigid models with two anisotropic sites joined at one end. For instance, Camp *et al* [136], and Lansac *et al* [137] have reported MC results for hard-core dimers formed by two spherocylinders with aspect ratios 1:1:2, and 1:1:5. Both papers studied the phase diagram for various apex (or aperture angles) finding N_u organizations for the larger angles, and isotropic fluids of interlocked dimers for smaller apertures.

Moving to attractive–repulsive potentials, similar results have been obtained with the GB models simulated by Memmer [138] and Neal and coworkers [139, 140] who performed NPT MC simulations of rigid bent-core dimers with aspect ratio 1:1:3 and various apex angles. The isotropic to N_u transition temperature was found to decrease when reducing the aperture angles. Even for this GB model, the N_u phase disappeared for an intermediate 170° apex angle. Interestingly, close to the nematic–smectic phase transition a spontaneous chirality symmetry breaking [141] was reported [138, 139, 142] to produce organizations related to those predicted by Lubensky and Radzihovsky [4], or observed by Görtz and Goodby [19]. The presence of a central transverse dipole [140] suppresses the N_u phase to give transitions from isotropic to smectic phases, and this is at variance with results from atomistic simulations (see later on). However, two terminal tilted dipoles embedded into the arms of a three GB bent-core model have been found

with MC simulations to stabilize the N_u phase with respect to smectic ordering [143] (see figure 10). No N_b organizations were observed for any of these systems. Interestingly, Clark and coworkers [144] have studied a three-site zig-zag model observing a rich polymorphism reminiscent of the dipolar bent-core models.

The multi-site models of rigidly connected LJ spherical sites simulated by Dewar and Camp [145, 89] produce results comparable the other bent-core models, but in this case the effect of a central transversal dipole seems to favour the N_u phase [89]. A five-site bent model with terminal flexible chains was also studied [89] to find that both smectic and N_u phases disappear. The effect of flexibility on the phase diagram, even though for linear chains, has been thoroughly studied instead by Galindo *et al* [146] with Gibbs ensemble MC simulations finding that the polymorphism is strongly affected by the non-rigid model.

To date, the most relevant simulation of multi-site bent-core mesogens has been that of Peláez and Wilson [61] who have performed the first MD simulation of a real N_b molecule [7, 8] using a full-atomistic potential including electrostatic interactions (see figure 11). In particular, the spontaneous ordering was observed cooling-down an isotropic sample, and Peláez and Wilson have given evidence regarding the formation of local ferroelectric domains in the N_b . Atomistic computer simulations are invaluable since they can help in studying some issues (e.g. flexibility), and the detailed effect of electrostatic charges as computed from *ab initio* calculations on specific molecules (see e.g. [147]) that would be difficult to tackle with theory or simpler potentials. For instance, in [61] the same atomistic model devoid of electrostatic charges (e.g. an overall transversal dipole moment) does not give a N_b phase but, upon cooling-down, a smectic one. This is again a confirmation that the subtle balance of anisotropic shape and interaction anisotropy is necessary in mesogenic molecules to stabilize the N_b phase.

The picture resulting from the published simulation work is that there is no general agreement regarding the optimal apex angle of bent-core mesogens for the stabilization of N_u phases, since the results also depend on other specific

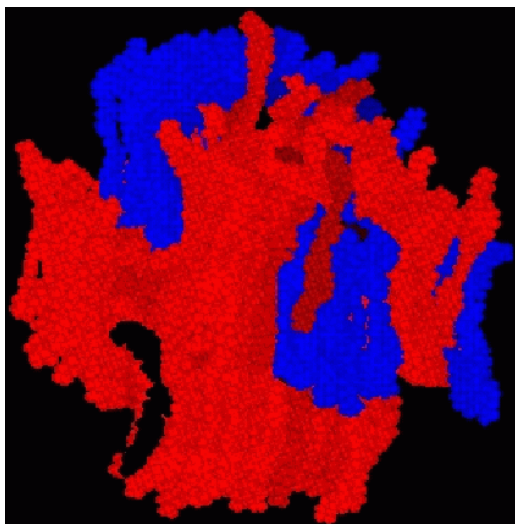


Figure 11. Snapshot of a bent-core N_b phase from the MD atomistic simulations of [61] showing the formation of ferroelectric domains with opposite polarity. (Image courtesy of Professor Mark R Wilson.)

model features. A certain bend seems to be necessary to observe a N_u , but the useful range of apertures may be fairly broad. A large apex angle generally favours the N_u with respect to layered organizations, while small values have given interlocked dimers (even though [47] reports N_b mesogens with 90° aperture). Certain intermediate apex angles destroy the N_u [136, 139, 140]. To provide a N_b phase a bent molecular shape is not sufficient, and other interaction terms (e.g. a suitable transverse dipole), or a certain amount of flexibility may be necessary to lower the symmetry, and/or the propensity to form ordered layers. To summarize, only atomistic simulations have been successful so far, and the lack of multi-site molecular models of N_b is a sign that we still do not know what are the minimal molecular features for obtaining these phases.

6. Mixtures

One of the possible pathways early recognized by Alben [24] as a candidate for finding N_b systems has been that of using a suitable mixture of rod- and disc-like mesogens fully miscible over the whole phase diagram. The rationale behind this strategy was that of creating a mixed nematic system where molecular symmetries would promote the independent self-alignment of the mesogens along two mutually orthogonal principal directors (one for the rods, and the other for the discs) producing a N_b system. This apparently simple picture is deceiving as to date the only published example of N_b mixture is not thermotropic but lyotropic [148], and even these findings have been subject to discussion and challenges [149, 150]. To our knowledge, no computer simulations have been conducted on lyotropic systems, possibly due to their complexity, but theoretical models strongly hint polydispersity [69], and rod-disc shape interconversion [151] as possible mechanisms underlying the N_b behaviour even for thermotropic mesogens.

In particular, the paper of Ratón and Cuesta [69] reports theoretical phase diagrams showing stable N_b mixtures of uniaxial board-like mesogens with 1:1:5, and 5:5:1 aspect ratios (see figure 6), and a Gaussian distribution of shape.

In spite of the neatness of the suggestion of Alben [24], the practical realization of a thermotropic N_b phase of mixed rod- and disc-like molecules has always proved to be difficult both on experimental [26] and modelling grounds [152–160]. The common reason behind this failure is that when both orientational and positional degrees of freedom are considered the ordered phases show a thermodynamic tendency to demix.

It is not surprising that one of the few computer simulation successes has been that of lattice models, like those of Hashim *et al* [161] where phase separation could not take place by design. It should be noted anyway that when particle exchange moves between the two distinct interpenetrated sub-lattices were included, a phase separation also takes place in this model system.

Similarly, almost all attempts made to simulate a biaxial rod-disc mixtures with off-lattice models have lead to phase separations as long as an ordering transition from the isotropic (mixed) phase set in. The majority of studies have been performed with hard particles, like the Gibbs MC simulations of Allen, Frenkel and coworkers [25], where only extreme aspect ratios (1:1:15 with 15:15:1, and 1:1:20 with 20:20:1) appeared to be compatible with a mixed phase of rod- and disc-like particles. Camp and Allen [162] have also simulated fluid mixtures of hard uniaxial ellipsoids with smaller aspect ratios 1:1:10 and 10:10:1 and composition 0.5 and 0.6. They have observed I, N, and N_b , and but have not explored higher density regions of the phase diagram to address the competition between an N_b phase and demixing. One possibility to overcome these difficulties would be that of enhancing specific rod-disc interactions [152, 154]. The aspect ratios typical of standard mesogens, when considered as hard rigid particles, are predicted by all theoretical models to produce demixing. This behaviour has been explained in terms of entropic effects related to the excluded volume of the mesogens, and their ratio [163, 164]. The mixed ordered state has higher mixing and orientational entropies, while the separated phases gain in translational entropy. As it turns out, for systems without extremely large aspect ratios, the latter contribution is dominant.

By using a theoretical model Camp and Allen [162] predict a symmetric phase diagram with respect to mole fraction, but forecast that using additional virial coefficients this may become asymmetrical. This has been specifically addressed by Vanakaras *et al* [165] who have studied, by MC simulations and theory, phase separation in mixtures of perfectly aligned hard boards with rod-like shape but different aspect ratios. The idea the three authors propose is that of disfavouring the smectic and crystal phases by mixing particles with similar breadth and width, but incommensurable lengths. This strategy also enhances miscibility and lowers the minimum aspect ratio necessary to stabilize an N_b phase. Another relevant result is that the most favourable concentration for observing an N_b system is not the standard equimolar, but instead a 0.3 mole fraction of the longer mesogen (see figure 12).

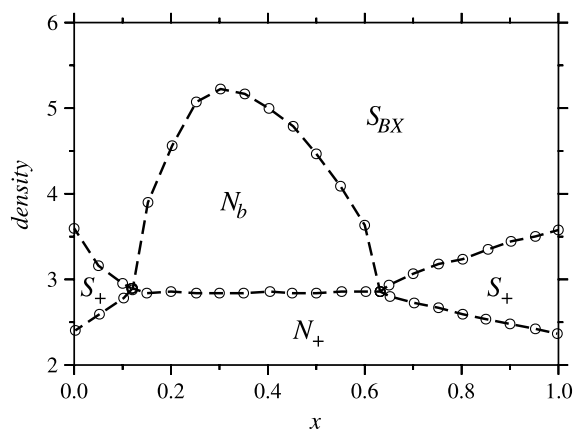


Figure 12. Phase diagram, with respect to the dimensionless reduced density and mole fraction x of the longer mesogen, for a two-component mixture of hard-body biaxial board-like particles from the theoretical model of [165]. All equilibrium curves correspond to second-order transitions. (Adapted from figure 3 of [165].)

Besides using specific rod–disc interactions, another strategy for overcoming phase separation in mixtures may be that of using molecules with both rod- and disc-like mesogenic units joined by a flexible alkyl spacer (the so-called shape amphiphiles [166]). Following the work of Fletcher and Luckhurst [167], other molecular design concepts have been explored [167, 168, 166, 169–171] obtaining N_u but not N_b . So far, the mesogen with closest resemblance to the theoretical concept of joined mesogens is the one of Kouwer and Mehl [169]. Relying on this picture, Bates and Luckhurst [172] have characterized extensively a lattice model with rods and discs at the same sites (completely overlapped sub-lattices) by using MC simulations. The effect of a flexible spacer has been modelled via a coupling between the rod and the disc at the same lattice site, to find from the simulations that the shape of the two-component phase diagram is quite modified by the strength of this interaction. Unfortunately, this lattice model with overlapped sites can not account for the segregation effects of rod and disc moieties observed experimentally [171, 170] and which may be interesting for other nanotechnological applications (although not for N_b). Rod–disc dimeric systems have also been studied with theoretical models [173], and experiments in presence of an electric [40] or magnetic [174] field. In the case of [174] a uniaxial discotic nematic was induced, while in [40] a field-induced N_b phase was achieved.

Considering all the previous modelling work, the findings of Apreutesei and Mehl [27] of completely miscible disc- and rod-shaped mesogens in the nematic phase are quite important since they are at variance with the theoretical predictions which always doom the mixed phase with the nemesis of demixing.

Attractive–repulsive off-lattice models have received small attention, and the question which choice of shape and interaction biaxialities can prevent demixing still has no answer. Also the effect of weak bonds between unlike particles [154] and flexibility have not been studied in detail by theory, and a model of miscible rod- and disc-like mesogens

with monodisperse shape distributions has never been put forward.

7. Concluding remarks

The field of N_b phases still poses many unanswered theoretical and practical questions, and besides experimental investigations there is large scope for modelling and computer simulations. Taking into account recent experimental results, our current view of the N_b phase, mostly derived from early theoretical and simulation models, might be too narrow and idealized. Rather than considering as N_b only nematic LC systems with spontaneous macroscopic biaxial ordering, we might broaden the classification to also include overall uniaxial systems with local biaxial or polar cybotactic clusters which could be converted into a N_b by a suitable weak perturbation, like a surface treatment or an external field.

If the nature of N_b organization is still not completely unravelled, on the molecular side we still do not know exactly what are the minimal features necessary to be accounted for in model potentials for obtaining an N_b phase. Electrostatic interactions, flexibility, and polydispersity are important ingredients which, along with shape anisotropy, may help in stabilizing N_b phases against freezing, layering or demixing.

Besides contributing in clarifying these issues, the future impact of N_b computer simulations relies on their ability in helping chemists to design candidate mesogens (e.g. by using atomistic models), to predict LC properties prior to the actual synthesis, and in contributing to the design of model devices (e.g. with lattice and molecular models) in view of a technological deployment of N_b materials. The achievement of these ambitious goals will require improved model potentials, predicting mesoscopic coefficients in bulk and confined environments, and devising efficient procedures for virtual computer experiments of responses.

Acknowledgments

We thank MUR, INSTM, and EU–STREP ‘*Biaxial Nematic Devices*’ (BIND) FP7-216025 for funding.

References

- [1] Luckhurst G R 2004 Liquid crystals: a missing phase found at last? *Nature* **430** 413–4
- [2] Freiser M J 1970 Ordered states of a nematic liquid *Phys. Rev. Lett.* **24** 1041–3
- [3] Straley J P 1974 Ordered phases of a liquid of biaxial particles *Phys. Rev. A* **10** 1881–7
- [4] Lubensky T C and Radzihovsky L 2002 Theory of bent-core liquid-crystal phases and phase transitions *Phys. Rev. E* **66** 031704
- [5] Berardi R, Ricci M and Zannoni C 2001 Ferroelectric nematic and smectic liquid crystals from tapered molecules *ChemPhysChem* **2** 443–7
- [6] Acharya B R, Primak A, Dingemans T J, Samulski E T and Kumar S 2003 The elusive thermotropic biaxial nematic phase in rigid bent-core molecules *Pramana* **61** 231–8
- [7] Acharya B R, Primak A and Kumar S 2004 Biaxial nematic phase in bent-core thermotropic mesogens *Phys. Rev. Lett.* **92** 145506

- [8] Madsen L A, Dingemans T J, Nakata M and Samulski E T 2004 Thermotropic biaxial nematic liquid crystals *Phys. Rev. Lett.* **92** 145505
- [9] Merkel K, Kocot A, Vij J K, Korlacki R, Mehl G H and Meyer T 2004 Thermotropic biaxial nematic phase in liquid crystalline organo-siloxane tetrapodes *Phys. Rev. Lett.* **93** 237801
- [10] Figueirinhas J L, Cruz C, Filip D, Feio G, Ribeiro A C, Frère Y, Meyer T and Mehl G H 2005 Deuterium NMR investigation of the biaxial nematic phase in an organosiloxane tetrapode *Phys. Rev. Lett.* **94** 107802
- [11] Luckhurst G R 2001 Biaxial nematic liquid crystals: fact or fiction? *Thin Solid Films* **393** 40–52
- [12] Praefcke K 2001 Can thermotropic biaxial nematics be made real? *Mol. Cryst. Liq. Cryst.* **364** 15–24
- [13] Chandrasekhar S 1993 *Liquid Crystals* 2nd edn (Cambridge: Cambridge University Press)
- [14] Luckhurst G R 2005 V-shaped molecules: new contenders for the biaxial nematic phase *Angew. Chem. Int. Edn* **44** 2834–6
- [15] Galerne Y 1997 Vanishing disclination lines in lyotropic biaxial nematics *Mol. Cryst. Liq. Cryst.* **292** 103–12
- [16] Galerne Y 1998 Characterization of biaxial nematic phases in the thermotropic liquid crystals *Mol. Cryst. Liq. Cryst.* **323** 211–29
- [17] Galerne Y 2006 Comment on ‘Thermotropic biaxial nematic liquid crystals’ *Phys. Rev. Lett.* **96** 219803
- [18] Vanakaras A G and Photinos D J 2008 Thermotropic biaxial nematic liquid crystals: spontaneous or field stabilized? *J. Chem. Phys.* **128** 154512
- [19] Görtz V and Goodby J W 2005 Enantioselective segregation in achiral nematic liquid crystals *Chem. Commun.* 3262–4
- [20] Präsang C, Whitwood A C and Bruce D W 2008 Spontaneous symmetry-breaking in halogen-bonded, bent-core liquid crystals: observation of a chemically driven Iso–N–N* phase sequence *Chem. Commun.* 2137–9
- [21] Biscarini F, Chiccoli C, Pasini P, Semeria F and Zannoni C 1995 Phase diagram and orientational order in a biaxial lattice model: a Monte Carlo study *Phys. Rev. Lett.* **75** 1803–6
- [22] Allen M P 1990 Computer-simulation of a biaxial liquid-crystal *Liq. Cryst.* **8** 499–511
- [23] Berardi R and Zannoni C 2003 Biaxial discotic Gay–Berne mesogens and biaxial nematics *Mol. Cryst. Liq. Cryst.* **396** 177–86
- [24] Alben R 1973 Liquid crystal phase transitions in mixtures of rodlike and platelike molecules *J. Chem. Phys.* **59** 4299–304
- [25] Camp P J, Allen M P, Bolhuis P G and Frenkel D 1997 Demixing in hard ellipsoid rod–plate mixtures *J. Chem. Phys.* **106** 9270–5
- [26] van der Kooij F M and Lekkerkerker H N W 2000 Liquid-crystalline phase behavior of a colloidal rod–plate mixture *Phys. Rev. Lett.* **84** 781–4
- [27] Apreutesei D and Mehl G H 2006 Completely miscible disc and rod shaped molecules in the nematic phase *Chem. Commun.* 609–11
- [28] Bruce D W 2004 Towards the biaxial nematic phase through molecular design *Chem. Record* **4** 10–22
- [29] Ros M B, Serrano J L, de la Fuente M R and Folcia C L 2005 Banana-shaped liquid crystals: a new field to explore *J. Mater. Chem.* **15** 5093–8
- [30] Bruce D W, Goodby J W, Sambles J R and Coles H J 2006 Introduction: new directions in liquid crystal science *Phil. Trans. R. Soc. A* **364** 2567–71
- [31] Pelletier O, Bourgaux C, Diat O, Davidson P and Livage J 1999 A biaxial nematic gel phase in aqueous vanadium pentoxide suspensions *Eur. Phys. J. B* **12** 541–6
- [32] Zannoni C 2001 Molecular design and computer simulations of novel mesophases *J. Mater. Chem.* **11** 2637–46
- [33] Care C M and Cleaver D J 2005 Computer simulation of liquid crystals *Rep. Prog. Phys.* **68** 2665–700
- [34] Wilson M R 2005 Progress in computer simulations of liquid crystals *Int. Rev. Phys. Chem.* **24** 421–55
- [35] Wilson M R 2007 Molecular simulation of liquid crystals: progress towards a better understanding of bulk structure and the prediction of material properties *Chem. Soc. Rev.* **36** 1881–8
- [36] Pasini P and Zannoni C (ed) 1999 *Advances in the Computer Simulation of Liquid Crystals* 1st edn (Dordrecht: Kluwer)
- [37] Bruce D W, Luckhurst G R and Photinos D J 1998 Discussion. Oxford workshop on biaxial nematic liquid crystals, December 1996 *Mol. Cryst. Liq. Cryst.* **323** 261–93
- [38] Taylor M P and Herzfeld J 1991 Nematic and smectic order in a fluid of biaxial hard particles *Phys. Rev. A* **44** 3742–51
- [39] Francescangeli O, Stanic V, Torgova S I, Strigazzi A, Scaramuzza N, Ferrero C, Dolbnya I P, Weiss T M, Berardi R, Muccioli L, Orlandi S and Zannoni C 2008 Ferroelectric-like response in a fluid nematic phase, in preparation
- [40] Jeong K-U, Jing A J, Mansdorf B, Graham M J, Yang D-K, Harris F W and Cheng S Z D 2007 Biaxial molecular arrangement of rod–disc molecule under an electric field *Chem. Mater.* **19** 2921–3
- [41] Torgova S I, Geivandova T A, Francescangeli O and Strigazzi A 2003 Banana-shaped 1,2,4-oxadiazoles *Pramana* **61** 239–48
- [42] Chiccoli C, Feruli I, Lavrentovich O D, Pasini P, Shiyonovskii S V and Zannoni C 2002 Topological defects in schlieren textures of biaxial and uniaxial nematics *Phys. Rev. E* **66** 030701
- [43] Picken S 2001 Measurements and values for selected order parameters *Physical Properties of Liquid Crystals: Nematics* ed D A Dunmur, A Fukuda and G R Luckhurst (London: Institution of Electrical Engineers (INSPEC)) chapter 2.2, p 89
- [44] Acharya B R, Kang S-W and Kumar S 2008 X-ray diffraction from the uniaxial and biaxial nematic phases of bent-core mesogens *Liq. Cryst.* **35** 109–18
- [45] Southern C D, Brimicombe P D, Siemianowski S D, Jaradat S, Roberts N, Görtz V, Goodby J W and Gleeson H F 2008 Thermotropic biaxial nematic order parameters and phase transitions deduced by Raman scattering *Europhys. Lett.* **82** 56001
- [46] Madsen L A *et al* 2006 Madsen Reply *Phys. Rev. Lett.* **96** 219804
- [47] Lehmann M, Kang S-W, Köhn C, Haseloh S, Kolb U, Schollmeyer D, Wang Q B and Kumar S 2006 Shape-persistent V-shaped mesogens-formation of nematic phases with biaxial order *J. Mater. Chem.* **16** 4326–34
- [48] Severing K and Saalwächter K 2004 Biaxial nematic phase in a thermotropic liquid-crystalline side-chain polymer *Phys. Rev. Lett.* **92** 125501
- [49] Hessel F and Finkelmann H 1986 Optical biaxiality of nematic LC-side chain polymers with laterally attached mesogenic groups *Polym. Bull.* **15** 349–52
- [50] Prasad V, Kang S-W, Suresh K A, Joshi L, Wang Q and Kumar S 2005 Thermotropic uniaxial and biaxial nematic and smectic phases in bent-core mesogens *J. Am. Chem. Soc.* **127** 17224–7
- [51] Vissenberg M C J M, Stallinga S and Vertogen G 1997 Generalized Landau–de Gennes theory of uniaxial and biaxial nematic liquid crystals *Phys. Rev. E* **55** 4367–77
- [52] Chiccoli C, Pasini P, Semeria F and Zannoni C 1999 A detailed Monte Carlo investigation of the tricritical region of a biaxial liquid crystal system *Int. J. Mod. Phys. C* **10** 469–76
- [53] Berardi R and Zannoni C 2000 Do thermotropic biaxial nematics exist? A Monte Carlo study of biaxial Gay–Berne particles *J. Chem. Phys.* **113** 5971–9

- [54] Cordoyiannis G, Apreutesei D, Mehl G H, Glorieux C and Thoen J 2008 High-resolution calorimetric study of a liquid crystalline organo-siloxane tetrapode with a biaxial nematic phase *Phys. Rev. E* **78** 011708
- [55] Lee J-H, Lim T-K, Kim W-T and Jin J-I 2007 Dynamics of electro-optical switching processes in surface stabilized biaxial nematic phase found in bent-core liquid crystal *J. Appl. Phys.* **101** 034105
- [56] Barberi R, Ciuchi F, Lombardo G, Bartolino R and Durand G E 2004 Time resolved experimental analysis of the electric field induced biaxial order reconstruction in nematics *Phys. Rev. Lett.* **93** 137801
- [57] Rosso R 2007 Orientational order parameters in biaxial nematics: polymorphic notation *Liq. Cryst.* **34** 737–48
- [58] Zannoni C 1979 Distribution functions and order parameters *The Molecular Physics of Liquid Crystals* ed G R Luckhurst and G W Gray (London: Academic) pp 51–83
- [59] Zannoni C 1985 Quantitative description of orientational order: rigid molecules *Nuclear Magnetic Resonance of Liquid Crystals* ed J W Emsley (Dordrecht: Reidel) chapter 1, pp 1–34
- [60] Berardi R, Muccioli L and Zannoni C 2004 Can nematic transitions be predicted by atomistic simulations? A computational study of the odd–even effect *ChemPhysChem* **5** 104–11
- [61] Peláez J and Wilson M R 2006 Atomistic simulations of a thermotropic biaxial liquid crystal *Phys. Rev. Lett.* **97** 267801
- [62] Biscarini F, Chiccoli C, Pasini P and Zannoni C 1991 On a simple model of ferroelectricity in liquid crystals *J. Non-Cryst. Solids* **131–133** 1190–3
- [63] Biscarini F, Zannoni C, Chiccoli C and Pasini P 1991 Head–tail asymmetry and ferroelectricity in uniaxial liquid crystals *Mol. Phys.* **73** 439–61
- [64] Berggren E, Tarroni R and Zannoni C 1993 Rotational diffusion of uniaxial probes in biaxial liquid crystal phases *J. Chem. Phys.* **99** 6180–200
- [65] Berggren E and Zannoni C 1995 Rotational diffusion of biaxial probes in biaxial liquid crystal phases *Mol. Phys.* **85** 299–333
- [66] Luckhurst G R, Zannoni C, Nordio P L and Segre U 1975 A molecular field theory for uniaxial nematic liquid crystals formed by non-cylindrically symmetric molecules *Mol. Phys.* **30** 1345–58
- [67] Sonnet A M, Virga E G and Durand G E 2003 Dielectric shape dispersion and biaxial transitions in nematic liquid crystals *Phys. Rev. E* **67** 061701
- [68] Longa L, Pajak G and Wydro T 2007 Stability of biaxial nematic phase for systems with variable molecular shape anisotropy *Phys. Rev. E* **76** 011703
- [69] Martínez-Ratón Y and Cuesta J A 2002 Enhancement by polydispersity of the biaxial nematic phase in a mixture of hard rods and plates *Phys. Rev. Lett.* **89** 185701
- [70] Biscari P, Napoli G and Turzi S 2006 Bulk and surface biaxiality in nematic liquid crystals *Phys. Rev. E* **74** 031708
- [71] Bisi F, Romano S and Virga E G 2007 Uniaxial rebound at the nematic biaxial transition *Phys. Rev. E* **75** 041705
- [72] Bisi F, Virga E G Jr, Gartland E C, De Matteis G, Sonnet A M and Durand G E 2006 Universal mean-field phase diagram for biaxial nematics obtained from a minimax principle *Phys. Rev. E* **73** 051709
- [73] Longa L, Grzybowski P, Romano S and Virga E G 2005 Minimal coupling model of the biaxial nematic phase *Phys. Rev. E* **71** 051714
- [74] Longa L and Pajak G 2005 Luckhurst–Romano model of thermotropic biaxial nematic phase *Liq. Cryst.* **32** 1409–17
- [75] De Matteis G, Romano S and Virga E G 2005 Bifurcation analysis and computer simulation of biaxial liquid crystals *Phys. Rev. E* **72** 041706
- [76] Allender D and Longa L 2008 Landau–de Gennes theory of biaxial nematics reexamined *Phys. Rev. E* **78** 011704
- [77] Chandrasekhar S, Nair G G, Shankar Rao D S, Prasad S K, Praefcke K and Blunk D 1998 A thermotropic biaxial nematic liquid crystal *Curr. Sci.* **75** 1042–6
- [78] Lavrentovich O D, Pasini P, Zannoni C and Zumer S (ed) 2001 *Defects in Liquid Crystals: Computer Simulations, Theory and Experiments* (Dordrecht: Kluwer)
- [79] Chiccoli C, Pasini P, Feruli I and Zannoni C 2005 Biaxial nematic droplets and their optical textures: a lattice model computer simulation study *Mol. Cryst. Liq. Cryst.* **441** 319–28
- [80] Priezjev N V and Pelcovits R A 2002 Coarsening dynamics of biaxial nematic liquid crystals *Phys. Rev. E* **66** 051705
- [81] Saupe A 1981 Elastic and flow properties of biaxial nematics *J. Chem. Phys.* **75** 5118–24
- [82] Brand H and Pleiner H 1981 Hydrodynamics of biaxial discotics *Phys. Rev. A* **24** 2777–88
- [83] Kini U D 1984 Isothermal hydrodynamics of orthorhombic nematics *Mol. Cryst. Liq. Cryst.* **108** 71–91
- [84] Kini U D 1984 Isothermal hydrodynamics of biaxial nematics *Mol. Cryst. Liq. Cryst.* **112** 265–84
- [85] Kini U D and Chandrasekhar S 1989 Elastic constants of a biaxial nematic liquid crystal *Physica A* **156** 364–75
- [86] Singh Y, Rajesh K, Menon V J and Singh S 1994 Molecular theory of elastic constants of liquid crystals. II. Application to the biaxial nematic phase *Phys. Rev. E* **49** 501–12
- [87] Kapanowski A 1997 Statistical theory of elastic constants of biaxial nematic liquid crystals *Phys. Rev. E* **55** 7090–104
- [88] Longa L, Stelzer J and Dunmur D 1998 Density functional approach to study the elastic constants of biaxial nematic liquid crystals *J. Chem. Phys.* **109** 1555–66
- [89] Dewar A and Camp P J 2005 Dipolar interactions, molecular flexibility, and flexoelectricity in bent-core liquid crystals *J. Chem. Phys.* **123** 174907
- [90] Harden J, Mbanga B, Éber N, Fodor-Csorba K, Sprunt S, Gleeson J T and Jákli A 2006 Giant flexoelectricity of bent-core nematic liquid crystals *Phys. Rev. Lett.* **97** 157802
- [91] Fiałkowski M 1998 Viscous properties of nematic liquid crystals composed of biaxial molecules *Phys. Rev. E* **58** 1955–66
- [92] Chiccoli C, Lavrentovich O D, Pasini P and Zannoni C 1997 Monte Carlo simulations of stable point defects in hybrid nematic films *Phys. Rev. Lett.* **79** 4401–4
- [93] Talbot J, Kivelson D, Allen M P, Evans G T and Frenkel D 1990 Structure of the hard ellipsoid fluid *J. Chem. Phys.* **92** 3048–57
- [94] Frenkel D 1988 Structure of hard-core models for liquid crystals *J. Phys. Chem.* **92** 3280–4
- [95] Bolhuis P and Frenkel D 1997 Tracing the phase boundaries of hard spherocylinders *J. Chem. Phys.* **106** 666–87
- [96] McGrother S C, Williamson D C and Jackson G 1996 A re-examination of the phase diagram of hard spherocylinders *J. Chem. Phys.* **104** 6755–71
- [97] Gay J G and Berne B J 1981 Modification of the overlap potential to mimic a linear site–site potential *J. Chem. Phys.* **74** 3316–9
- [98] Berne B J and Pechukas P 1972 Gaussian model potentials for molecular interactions *J. Chem. Phys.* **56** 4213–6
- [99] Ayton G and Patey G N 1995 A generalized Gaussian overlap model for fluids of anisotropic particles *J. Chem. Phys.* **102** 9040–7
- [100] Berardi R, Fava C and Zannoni C 1995 A generalized Gay–Berne intermolecular potential for biaxial particles *Chem. Phys. Lett.* **236** 462–8

- [101] Berardi R, Fava C and Zannoni C 1998 A Gay–Berne potential for dissimilar biaxial particles *Chem. Phys. Lett.* **297** 8–14
- [102] Everaers R and Ejtehadi M R 2003 Interaction potentials for soft and hard ellipsoids *Phys. Rev. E* **67** 041710
- [103] Zheng X and Palffy-Muhoray P 2007 Distance of closest approach of two arbitrary hard ellipses in two dimensions *Phys. Rev. E* **75** 061709
- [104] Cleaver D J, Care C M, Allen M P and Neal M P 1996 Extension and generalization of the Gay–Berne potential *Phys. Rev. E* **54** 559–67
- [105] Babadi M, Everaers R and Ejtehadi M R 2006 Coarse-grained interaction potentials for anisotropic molecules *J. Chem. Phys.* **124** 174708
- [106] Babadi M and Ejtehadi M R 2007 An exact treatment of ellipsoid–substrate interactions *Europhys. Lett.* **77** 23002
- [107] Paramonov L and Yaliraki S N 2005 The directional contact distance of two ellipsoids: Coarse-grained potentials for anisotropic interactions *J. Chem. Phys.* **123** 194111
- [108] Perram J W and Wertheim M S 1985 Statistical mechanics of hard ellipsoids. I. Overlap algorithm and the contact function *J. Comput. Phys.* **58** 409–16
- [109] Plimpton S J 1995 Fast parallel algorithms for short-range molecular dynamics *J. Comput. Phys.* **117** 1–9
- [110] Plimpton S J 2008 LAMMPS molecular dynamics simulator <http://lammps.sandia.gov/>
- [111] Cheung D L, Clark S J and Wilson M R 2002 Parametrization and validation of a force field for liquid-crystal forming molecules *Phys. Rev. E* **65** 051709
- [112] Wang Z Q, Lupo J A, Patnaik S and Pachter R 2001 Large scale molecular dynamics simulations of a 4-n-pentyl-4'-cyanobiphenyl (5CB) liquid crystalline model in the bulk and as a droplet *Comput. Theor. Polym. Sci.* **11** 375–87
- [113] McDonald A J and Hanna S 2004 Atomistic computer simulation of terraced wetting of model 8CB molecules at crystal surfaces *Mol. Cryst. Liq. Cryst.* **413** 135–44
- [114] McDonald A J and Hanna S 2006 Atomistic simulation of a model liquid crystal *J. Chem. Phys.* **124** 164906
- [115] Tiberio G, Muccioli L, Berardi R and Zannoni C 2008 Towards *in silico* liquid crystals. Realistic transition temperatures and physical properties for n-cyanobiphenyls via molecular dynamics simulations *ChemPhysChem* at press doi:10.1002/cphc.200800231
- [116] Cacelli I, De Gaetani L, Prampolini G and Tani A 2007 Liquid crystal properties of the n-alkyl-cyanobiphenyl series from atomistic simulations with *ab initio* derived force fields *J. Phys. Chem. B* **111** 2130–7
- [117] Luckhurst G R and Romano S 1980 Computer simulation studies of anisotropic systems *Mol. Phys.* **40** 129–39
- [118] De Matteis G and Virga E G 2005 Tricritical points in biaxial liquid crystal phases *Phys. Rev. E* **71** 061703
- [119] Neupane K, Kang S W, Sharma S, Carney D, Meyer T, Mehl G H, Allender D W, Kumar S and Sprunt S 2006 Dynamic light scattering study of biaxial ordering in a thermotropic liquid crystal *Phys. Rev. Lett.* **97** 207802
- [120] Romano S 2004 Computer simulation of a biaxial nematogenic model on a three-dimensional lattice and based on a recently proposed interaction potential *Physica A* **337** 505–19
- [121] Bates M A and Luckhurst G R 2005 Biaxial nematic phases and V-shaped molecules: a Monte Carlo simulation study *Phys. Rev. E* **72** 051702
- [122] Bates M A 2006 Influence of flexibility on the biaxial nematic phase of bent core liquid crystals: a Monte Carlo simulation study *Phys. Rev. E* **74** 061702
- [123] Romano S 2004 Computer simulation study of a biaxial nematogenic lattice model associated with a two-dimensional lattice *Physica A* **339** 491–510
- [124] Romano S 2004 Computer simulation study of a biaxial nematogenic lattice model associated with a three-dimensional lattice and involving dispersion interactions *Physica A* **339** 511–30
- [125] Romano S 2004 Mean-field and computer simulation study of a biaxial nematogenic lattice model mimicking shape amphiphilicity *Phys. Lett. A* **333** 110–9
- [126] Camp P J and Allen M P 1997 Phase diagram of the hard biaxial ellipsoid fluid *J. Chem. Phys.* **106** 6681–8
- [127] McBride C and Lomba E 2007 Hard biaxial ellipsoids revisited: numerical results *Fluid Phase Equilib.* **255** 37–45
- [128] Chandrasekhar S 1985 Relation between molecular structure and liquid crystalline properties *Mol. Cryst. Liq. Cryst.* **124** 1–20
- [129] Ciccotti G, Jacucci G and McDonald I R 1979 Thought-experiments by molecular dynamics *J. Stat. Phys.* **21** 1–22
- [130] Berardi R, Muccioli L and Zannoni C 2008 Field response and switching times in biaxial nematics *J. Chem. Phys.* **128** 024905
- [131] Cîrcu V, Gibbs T J K, Omnès L, Horton P N, Hursthouse M B and Bruce D W 2006 Orthometallated palladium(II) imine complexes as candidate materials for the biaxial nematic phase. Crystal and molecular structure of three palladium imine complexes *J. Mater. Chem.* **16** 4316–25
- [132] Sarman S 1996 Molecular dynamics of biaxial nematic liquid crystals *J. Chem. Phys.* **104** 342–50
- [133] Sarman S 2000 Biaxial nematic liquid crystal phases of a soft ellipsoid-string fluid *Phys. Chem. Chem. Phys.* **2** 3831–9
- [134] Sarman S 1998 Molecular dynamics calculation of the viscosities of biaxial nematic liquid crystals *Int. J. Thermophys.* **19** 1073–83
- [135] Sarman S 1997 Shear flow simulations of biaxial nematic liquid crystals *J. Chem. Phys.* **107** 3144–58
- [136] Camp P J, Allen M P and Masters A J 1999 Theory and computer simulation of bent-core molecules *J. Chem. Phys.* **111** 9871–81
- [137] Lansac Y, Maiti P K, Clark N A and Glaser M A 2003 Phase behavior of bent-core molecules *Phys. Rev. E* **67** 011703
- [138] Memmer R 2002 Liquid crystal phases of achiral banana-shaped molecules: a computer simulation study *Liq. Cryst.* **29** 483–96
- [139] Johnston S J, Low R J and Neal M P 2002 Computer simulation of apolar bent-core and rodlike molecules *Phys. Rev. E* **65** 051706
- [140] Johnston S J, Low R J and Neal M P 2002 Computer simulation of polar bent-core molecules *Phys. Rev. E* **66** 061702
- [141] Heppke G and Moro D 1998 Liquid crystals: chiral order from achiral molecules *Science* **279** 1872–3
- [142] Xu J, Selinger R L B, Selinger J V and Shashidhar R 2001 Monte Carlo simulation of liquid-crystal alignment and chiral symmetry-breaking *J. Chem. Phys.* **115** 4333–8
- [143] Orlandi S, Berardi R, Stelzer J and Zannoni C 2006 A Monte Carlo study of the mesophases formed by polar bent-shaped molecules *J. Chem. Phys.* **124** 124907
- [144] Maiti P K, Lansac Y, Glaser M A and Clark N A 2004 Entropy-stabilized smectic C phase in a system of zigzag-shaped molecules *Phys. Rev. Lett.* **92** 025501
- [145] Dewar A and Camp P J 2004 Computer simulations of bent-core liquid crystals *Phys. Rev. E* **70** 011704
- [146] Galindo A, Vega C, Sanz E, MacDowell L G, de Miguel E and Blas F J 2004 Computer simulation study of the global phase behavior of linear rigid Lennard-Jones chain molecules: comparison with flexible models *J. Chem. Phys.* **120** 3957–68

- [147] Cacelli I and Prampolini G 2005 DFT conformational study of banana-shaped mesogens *Chem. Phys.* **314** 283–90
- [148] Yu L J and Saupe A 1980 Observation of a biaxial nematic phase in potassium laurate-1-decanol-water mixtures *Phys. Rev. Lett.* **45** 1000–3
- [149] Henriques E F and Henriques V B 1997 Biaxial phases in polydisperse mean-field model solution of uniaxial micelles *J. Chem. Phys.* **107** 8036–40
- [150] Berejnov V, Cabuil V, Perzynski R and Raikher Yu 1998 Lyotropic system potassium laurate/1-decanol/water as a carrier medium for a ferromagnetic liquid crystal: phase diagram study *J. Phys. Chem. B* **102** 7132–8
- [151] Tolédano P and Figueiredo Neto A M 1994 Topological metamorphosis in the phase diagram of a liquid of biaxial micellar aggregates *Phys. Rev. Lett.* **73** 2216–9
- [152] Sharma S R, Palfy-Muhoray P, Bergersen B and Dunmur D A 1985 Stability of a biaxial phase in a binary mixture of nematic liquid crystals *Phys. Rev. A* **32** 3752–5
- [153] van der Kooij F M and Lekkerkerker H N W 2000 Liquid-crystal phases formed in mixed suspensions of rod- and platelike colloids *Langmuir* **16** 10144–9
- [154] Vanakaras A G, McGrother S C, Jackson G and Photinos D J 1998 Hydrogen-bonding and phase biaxiality in nematic rod–plate mixtures *Mol. Cryst. Liq. Cryst.* **323** 199–209
- [155] Vanakaras A G and Photinos D J 1997 Theory of biaxial nematic ordering in rod–disc mixtures revisited *Mol. Cryst. Liq. Cryst.* **299** 65–71
- [156] Chrzanowska A 1998 Bicritical points in a nematic rod–disc mixture *Phys. Rev. E* **58** 3229–36
- [157] Galindo A, Haslam A J, Varga S, Jackson G, Vanakaras A G, Photinos D J and Dunmur D A 2003 The phase behavior of a binary mixture of rodlike and disclike mesogens: Monte Carlo simulation, theory, and experiment *J. Chem. Phys.* **119** 5216–25
- [158] Cuetos A, Martínez-Haya B, Lago S and Rull L F 2007 Use of Parsons–Lee and Onsager theories to predict nematic and demixing behavior in binary mixtures of hard rods and hard spheres *Phys. Rev. E* **75** 061701
- [159] Varga S, Galindo A and Jackson G 2002 Phase behavior of symmetric rod–plate mixtures revisited: biaxiality versus demixing *J. Chem. Phys.* **117** 10412–24
- [160] Varga S, Purdy K, Galindo A, Fraden S and Jackson G 2005 Nematic–nematic phase separation in binary mixtures of thick and thin hard rods: results from Onsager-like theories *Phys. Rev. E* **72** 051704
- [161] Hashim R, Luckhurst G R, Prata F and Romano S 1993 Computer simulation studies of anisotropic systems. XXII. An equimolar mixture of rods and discs: a biaxial nematic? *Liq. Cryst.* **15** 283–309
- [162] Camp P J and Allen M P 1996 Hard ellipsoid rod–plate mixtures: Onsager theory and computer simulations *Physica A* **229** 410–27
- [163] Stroobants A and Lekkerkerker H N W 1984 Liquid crystal phase transitions in a solution of rodlike and disklike particles *J. Phys. Chem.* **88** 3669–74
- [164] Wensink H H, Vroege G J and Lekkerkerker H N W 2002 Biaxial versus uniaxial nematic stability in asymmetric rod–plate mixtures *Phys. Rev. E* **66** 041704
- [165] Vanakaras A G, Bates M A and Photinos D J 2003 Theory and simulation of biaxial nematic and orthogonal smectic phases formed by mixtures of board-like molecules *Phys. Chem. Chem. Phys.* **5** 3700–6
- [166] Date R W and Bruce D W 2003 Shape amphiphiles: mixing rods and disks in liquid crystals *J. Am. Chem. Soc.* **125** 9012–3
- [167] Fletcher I D and Luckhurst G R 1995 The synthesis and characterization of novel non-symmetric dimers with rod-like and disc-like mesogenic units *Liq. Cryst.* **18** 175–83
- [168] Hunt J J, Date R W, Timimi B A, Luckhurst G R and Bruce D W 2001 Toward the biaxial nematic phase of low molar mass thermotropic mesogens: substantial molecular biaxiality in covalently linked rod–disk dimers *J. Am. Chem. Soc.* **123** 10115–6
- [169] Kouwer P H J and Mehl G H 2003 Full miscibility of disk- and rod-shaped mesogens in the nematic phase *J. Am. Chem. Soc.* **125** 11172–3
- [170] Kouwer P H J and Mehl G H 2003 Multiple levels of order in linked disc-rod liquid crystals *Angew. Chem. Int. Edn* **42** 6015–8
- [171] Kouwer P H J, Pourzand J and Mehl G H 2004 Disc-shaped triphenylenes in a smectic organisation *Chem. Commun.* **1** 66–7
- [172] Bates M A and Luckhurst G R 2005 Biaxial nematics: computer simulation studies of a generic rod–disc dimer model *Phys. Chem. Chem. Phys.* **7** 2821–9
- [173] Vanakaras A G, Terzis A F and Photinos D J 2001 On the molecular requirements for the stabilisation of thermotropic biaxial ordering in rod–plate nematics *Mol. Cryst. Liq. Cryst.* **362** 67–78
- [174] Imrie C T, Lu Z, Picken S J and Yildirim Z 2007 Oligomeric rod–disc nematic liquid crystals *Chem. Commun.* **1245–7**
- [175] Stannarius R 2008 Comment on ‘Dynamics of electro-optical switching processes in surface stabilized biaxial nematic phase found in bent-core liquid crystal’ *J. Appl. Phys.* **104** 036104